Prabhat Kumar College, Contai

Department of Mathematics

4th Semester Mathematics (Hons) CBCS Pattern

Paper: C-10 **Time: 1 hours**

Answer any one :-

1. Prove that every finite integral domain is a field.
2. Prove that the characteristic of an integral domain R is either prime or zero.
3. Let R be a commutative ring with $1(\neq 0)$ and P be an ideal of R. Then prove that P is a proper prime ideal iff R/P is an integral domain.
4. Show that $\mathbb{Q}[\sqrt{2}] = \{a + b\sqrt{2} \in \mathbb{R} \mid a, b \in \mathbb{Q}\}$ is a subfield of the field $\mathbb{R}$.
5. Prove that if W_1 and W_2 are finite dimensional subspaces of a vector space V ,then the subspace $W_1 + W_2$ is finite dimensional and $\dim(W_1 + W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2)$.
6. Let V and W be vector spaces of equal (finite) dimension and let $T: V \to W$ be linear. Then prove that the following are equivalent.
   a) T is one-to-one.
   b) T is onto.