Prabhat Kumar College, Contai

Department of Mathematics

4th Semester Mathematics (Hons) CBCS Pattern

Paper: C-8 **Time: 1 hours**

Answer any one :-

1. Evaluate $\int_0^\infty \frac{x^{p-1}}{1+x} dx, 0 < p < 1$
2. Find the value of $\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{2}{n}\right) \ldots\ldots \Gamma\left(\frac{n-1}{n}\right)$ where n is an integer.
3. Define interval of convergence of a power series . Show that a power series can be differentiated term by term within the interval of convergence.
4. Use first mean value theorem to prove that

$$\frac{\pi}{2} \leq \int_0^{1/2} \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}} \leq \frac{\pi}{2} \frac{1}{\sqrt{\left(1-k^2/4\right)}}, k^2 < 1.$$

5. Give an example of a sequence of functions such that $\lim_{n\to\infty} \int_0^1 f_n(x)dx \neq \int_0^1 f(x)dx.$