Prabhat Kumar College, Contai

Department of Mathematics

4th Semester Mathematics (Hons) CBCS Pattern

Paper: C-9 **Time: 1 hours**

Answer any one :-

1. If $S \subset \mathbb{R}^2$ is a bounded and closed set and if f is continuous on S then show that f is bounded.
2. If f is a function defined in a deleted neighbourhood $N(x_0 - y_0) - \{(x_0 - y_0)\}$ of $(x_0 - y_0)$ and if $\lim_{(x,y)\to(x_0 - y_0)} f(x,y) = l$ then show that $f(x,y) \to l$ as $(x,y) \to (x_0 - y_0)$ through any path $y = \emptyset(x)$ such that $(x, \emptyset(x)) \to (x_0 - y_0)$.
3. Let $D \subset \mathbb{R}^2$ be an open set and $f: D \to \mathbb{R}$ and $(a,b) \in D$. If $f_{xy}(a,b)\ and\ f_{yx}(a,b)$ both are continuous at (a,b) then prove that $f_{xy}(a,b) = f_{yx}(a,b)$.
4. If $f(x,y) = \frac{xy(x^2-y^2)}{x^2+y^2}$, $(x,y) \neq (0,0)$ and $f(0,0) = 0$ then show that $f_{xy}(0,0) = f_{yx}(0,0)$. Also show that the function $f(x,y)$ does not satisfy the hypotheses of Schwarzs theorem.
5. Let $u = f(x,y)$ where $= r\cos\theta$, $y = r\sin\theta$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$.